R·I·T

2018 Imaging Science Ph.D. Qualifying Examination
June 1, 2018
9:00AM to 12:00PM

IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use AND STAY INSIDE THE BOX! Be sure to write your provided identification letter, the question number, and a sequential page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name on the supplied 5" x 8" paper containing your provided identification letter and place this in the small envelope, and then place this envelope along with your answer sheets in the large envelope.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: __________________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD

Core 1: Fourier Methods in Imaging. Answer TWO Questions from Questions 1-3

1. **Fourier Methods in Imaging (10 points).** Consider an imaging system that acts on 2-D spatial functions with quadratic-phase impulse response:

$$h[x, y; \lambda_0, z_1] = \exp\left[+i\pi \frac{x^2 + y^2}{\lambda_0 z_1}\right]$$

where λ_0 and z_1 are constant parameters each with dimensions of length. The impulse response has the form of an expanding paraboloid.

   (a) Evaluate the impulse response $w[x, y; \lambda_0, z_1]$ and the transfer function $W[\xi, \eta; \lambda_0, z_1]$ of the inverse filter for this system.

   (b) In the case where the input to the system is a converging paraboloid:

$$f_1[x, y] = \exp\left[-i\pi \frac{x^2 + y^2}{\lambda_0 z_1}\right]$$

   Evaluate the output amplitude $g_1[x, y; \lambda_0, z_1]$ and squared magnitude $|g_1[x, y; \lambda_0, z_1]|^2$ from the system.

   (c) Evaluate the output amplitude $g_2[x, y; \lambda_0, z_1]$ and squared magnitude $|g_2[x, y; \lambda_0, z_1]|^2$of the system if the input function is $f_1[x, y]$ truncated by a "square" aperture with sides of width b_0.

2. **Fourier Methods in Imaging (10 points).**
Consider an input function $f[x]$ where $0 \leq f \leq 1$. A nonlinear operator $\mathcal{N}$ is applied to this function:

$$\begin{aligned}
\mathcal{N}\{f[x]\} &= g[x] \\
&\equiv (1 - GAUS[f[x]]) \cdot RECT\left[f - \frac{1}{2}\right] \\
&= \left(1 - \exp\left[-\pi (f[x])^2\right]\right) \cdot RECT\left[f - \frac{1}{2}\right]
\end{aligned}$$

Examples of outputs are shown in the table:

$f[x]$	$g[x]$
2	$\left(1 - \exp\left[-\pi (2)^2\right]\right) \cdot RECT\left[2 - \frac{1}{2}\right] = 0$
1	$1 - GAUS[1] = 1 - e^{-\pi} \cong 0.957$
$\frac{1}{2}$	$1 - GAUS\left[\frac{1}{2}\right] = 1 - e^{-\frac{\pi}{2^2}} \cong 0.544$
0	$1 - GAUS[0] = 0$
-1	$\left(1 - \exp\left[-\pi (-1)^2\right]\right) \cdot RECT\left[-1 - \frac{1}{2}\right] = 0$

(a) Sketch $g[x]$ if the input function $f[x] = TRI[x]$.

(b) Evaluate $G[\xi]$ if the input is the biased cosine $f[x] = \frac{1}{2} + \frac{1}{2}\cos[2\pi x]$, which means that $0 \leq f \leq 1$. You may make reasonable approximations, but state what you use.

3. **Fourier Methods in Imaging (10 points).**
Consider the 2-D function:

$$f[x,y] = \delta[x] \cdot 1[y] + 1[x] \cdot \delta[y]$$

(a) (10%) Sketch $f[x,y]$ in some manner that shows its 2-D form.

(b) (25%) Evaluate and sketch $F[\xi,\eta] = \mathcal{F}_2\{f[x,y]\}$

(c) (10%) Sketch the function $f[x,y]$ after rotation about the origin by θ radians to create $\mathcal{R}_\theta\{f[x,y]\} \equiv r[x,y;\theta]$.

(d) (25%) Describe the difference between the 2-D spectra of $f[x,y]$ and $r[x,y;\theta]$, i.e., between $F[\xi,\eta]$ and $R[\xi,\eta;\theta]$.

(e) (30%) The spatial integral of the set of rotated replicas of $f[x,y]$ is:

$$\int_{\theta=-\frac{\pi}{2}}^{\theta=+\frac{\pi}{2}} r[x,y;\theta]\ d\theta = \frac{1}{\sqrt{x^2+y^2}}$$

Evaluate the spectrum $\mathcal{F}_2\left\{\frac{1}{\sqrt{x^2+y^2}}\right\}$ (HINT: Hankel transform)

Core 2: Optics. Answer TWO Questions from Questions 4-6

Optics Instructions: Each answer must be accompanied by supporting illustrations. Be sure to label the axes and all important characteristics in your sketch. Illegible work will not be graded. Partial credit will be awarded if it demonstrates a level of competency. Please use a straight edge ruler (or paper edge) and a single purpose calculator when needed. If you do not have one, please tell the proctor.

4. **Optics (10 points). Fraunhofer Diffraction.** The moon is $Z_m = 3.8 \times 10^8 m$ from the earth. You wish to direct a beam of wavelength $\lambda = 1\mu m$ from the earth to the moon, so that an Airy disk of radial size, R_m illuminates a small portion of the lunar surface.

   a) State how this may be achieved using only Fraunhofer diffraction (without a lens). You must provide a detailed sketch of the system. Also state any assumptions about Fraunhofer diffraction.

   b) Express the electric field at Z_m in terms of an integral over the electric field of the beam on the Earth. You MAY NOT use shorthand notation such as $FT[E(x)]$. Evaluate the integral by hand (you must show your work) and determine expressions for the characteristic angular beam width $\Delta\theta$ and Airy disk size R_m at the distance Z_m.

   c) If you design your Earth system to achieve $R_m = 20m$, will the far-field condition be satisfied? You must prove your answer (i.e., evaluate the diffraction length and compare it to Z_m).

5. **Optics (10 points). Imaging with the Eye.** When gazing at an object at the Near Point, the focal length of an eye is found to be $f_{np} = 1.85cm$.

   a) An object of height $h = 50\mu m$ is place at the Near Point of the eye, $Z_{np} = 25cm$. Determine the image height on the retina (expressed in μm). You must provide an instructive ray diagram to receive credit.

   b) Determine the incoherent cut-off frequency for part (a), assuming a pupil diameter $D_{pupil} = 4mm$ and a wavelength $\lambda = 0.5\mu m$. Express your answer in units of oscillations per mm.

   c) A magnifying lens of focal length $f_{ml} = 12cm$ is placed between the object and the eye, with the object located $11.5cm$ from the lens. The focal length of the eye for a far point is $f_{fp} = 2.0cm$. Assume the distance between the magnifying lens and the front of the eye is $1.0cm$. Determine the image height on the retina. You must provide an instructive ray diagram to receive credit.

6. **Optics (10 points). Paraxial Ray Tracing.** For each of the following, provide a detailed sketch, along with all calculations and derivations. To receive credit, YOU MUST PROVE each answer (not simply write a memorized result). Use the convention $y_2 = Ay_1 + B\theta_1$, $\theta_2 = Cy_1 + D\theta_1$.

   (a) Derive the $ABCD$ matrix for ray propagation over a distance d through a homogeneous isotropic medium.

   (b) Derive the $ABCD$ matrix for ray propagation across an interface between the first material having an index of refractive, n_1, and the second material having an index of refractive, n_2.

   (c) A Galilean telescope is comprised of a positive and negative lens of respective focal lengths $f_1 = f$ and $f_2 = -f/10$. The $ABCD$ matrix for a thin lens of focal length f_0 has elements $A = D = 1$, $B = 0$, and $C = -1/f_0$. Determine the system (lens-space-lens) $ABCD$ matrix for the telescope as a function of the lens separation distance, d. Use your result to determine an expression for d (in terms of f) that produces a collimated output beam when the incoming beam is also collimated. State whether your answer is reasonable.

Core 3: Human Vision. Answer TWO Questions from Questions 7-9

7. **Human Vision (10 Points).** Your team is working on a new head-mounted display (HMD) to be used by neurosurgeons to perform tele-operations on patients in operating rooms a few thousand km away. The display is to be binocular, driven by two high-resolution cameras on an operating robot controlled by the neurosurgeon.

   Describe the important parameters driven by the human visual system that you would consider in designing the HMD. Include in your description the parameter, the reason it is important, relevant values, and any considerations that should be taken into account. For example (briefly): Parameter 1 - color, which is important because color carries information in the context of an operation for describing tissue types, presence of blood, etc.. Relevant values - 3 color channels to drive the S, M, & L wavelength cone classes; $\lambda = \sim 400 - 700\,nm$ to cover the visible range. Considerations - some people are color blind or color-anomalous, i.e., missing one or more cone classes or the spectral response of one or more cone classes is shifted.

8. **Human Vision (10 Points).** The human visual system works over a huge range of illumination conditions. Describe the range and the mechanisms that allow this flexibility, including any trade-offs that are made to permit the large range.

   The human visual system also works over a large range of object distances. Describe the range and the mechanisms that allow this flexibility.

9. **Human Vision (10 Points).** A helicopter flies overhead close enough that you can see the pilot's face clearly. At that distance, you see that the helicopter is painted with a fine pattern of black and white lines, so you take out your custom CCD video camera and start recording video of the helicopter as it flies away.

   When you play the video back on a display with the same spatial and temporal resolution as the CCD sensor (i.e. it shows every pixel of every frame captured by the camera) you notice two interesting things. First, the main rotor appears to be stationary, not rotating. Second, while the pattern of black and white lines is clearly visible at the beginning of the video, as it flies away from your location, the contrast of the lines first fades out towards a gray, then reappears, but in an uneven, wavering pattern of much wider light and dark lines, then fades back to gray again. When you were looking directly at the helicopter the blades were clearly rotating, and while the lines eventually faded to a uniform gray, you never saw the wider line pattern before they faded to gray.

   i. What caused the two phenomena described above? Explain the origin of both effects, and describe what (if anything) you could determine about the characteristics of the helicopter given the technical specifications for your video camera.

   ii. Why didn't you experience the same phenomena when you were viewing the helicopter directly?

R·I·T

2018 Imaging Science Ph.D. Qualifying Examination
June 1, 2018
1:00PM to 4:00PM

IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use AND STAY INSIDE THE BOX! Be sure to write your provided identification letter, the question number, and a sequential page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name on the supplied 5" x 8" paper containing your provided identification letter and place this in the small envelope, and then place this envelope along with your answer sheets in the large envelope.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: ________________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD

Core 4: Radiometry, Answer TWO Questions from Questions 10-12

Several constants that you may or may not need in your solutions are provided here for your convenience:

Planck's Constant: $h = 6.626 \times 10^{-34} m^2 kg/s$
Speed of light: $c = 3 \times 10^8 m/s$
Charge of an electron: $e = 1.602 \times 10^{-19} coulomb$
Boltzmann Constant: $k = 1.381 \times 10^{-23}$ J/K
Approximate Solar Temperature: $T_{solar} = 5800K$
Approximate Room Temperature: $T_{room} = 294K$
Earth-Sun Distance: $D_{earth-sun} = 1.4960 \times 10^{11} m$.
Radius of the Sun: $R_{sun} = 6.955 \times 10^8 m$
Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8} wm^{-2} K^{-4}$
Absolute zero on Kelvin scale converted to Celsius: $-273.15°C$

10. **Radiometry (10 points).** You are comparing two different collimated, monochromatic light beams using the same detector. In each case, the beam strikes the detector perpendicular to the face of the detector. When the first beam is operating, it has a wavelength λ_1, and at the detector, it has a radiant flux $\Phi_1\,[W]$. When the second beam is operating, it has wavelength λ_2, and a radiant flux $\Phi_2\,[W]$ at the detector.

   a) (5 points). Find an expression for the percentage change in electron shot noise of the detector when the second beam is impinging on the detector, as seen in the Figure below, compared to the shot noise observed when the first beam is impinging on the detector.

   b) (2 points). Assume that the quantum efficiency of the detector is that shown in the curve with the solid line in the graph in the Figure. Let $\lambda_1 = 400nm$ for the first monochromatic beam and $\lambda_2 = 800\,nm$ for the second beam, and let the corresponding radiant flux of the two beams be $\Phi_1 = 0.08\,\mu W$ and $\Phi_2 = 0.32\,\mu W$. Use this information and the graph below to evaluate your answer to part (a) numerically.

   c) (3 points). For this part, suppose now that the flux of the two beams described in part (b) was recorded during a time interval $t = 1.6\,\mu s$. Suppose also that the only other significant source of noise is the read noise of the detector, and assume that the read noise remains the same for both beams with a constant value of $N_{read} = 620\,electrons$. Assume also that all other characteristics of the detector are the same as in part (b). With these assumptions, find the signal-to-noise ratio for each beam.

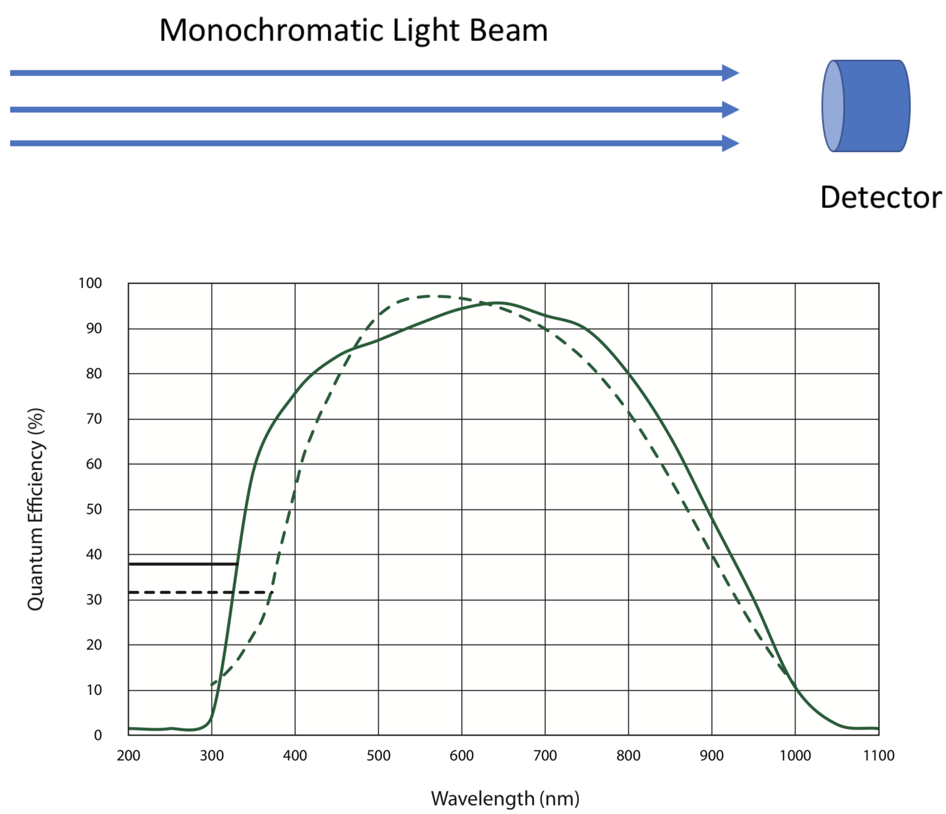


Figure for Problem 10: (Top) Light beam strikes the detector at normal incidence. (Bottom) Quantum efficiency vs wavelength (source Princeton Instruments, Inc.)

11. **Radiometry (10 Points).** An isotropic light source is suspended just below the surface of the water column in a shallow region. Assume that the light is a monochromatic point source with radiant flux $\Phi_0\,[W]$ and wavelength λ. Assume that there are no other sources of light in the set-up portrayed in the Figure. As shown in the Figure below, you attach a water-proof detector, with responsivity $R\,[A\,W^{-1}]$, to a rigid pole and rod assembly so that the detector is positioned directly below the light source and looking up at the light source at a depth d below the water surface. At that depth, the detector measures a signal $S(d)\;[A]$, which through the detector calibration, provides an estimate of the flux arriving at the detector at that depth: $\Phi(d)\;[W]$.

   a) (3 points). Find a mathematical expression for the extinction coefficient of the water column, using one or more of the given variables.

   b) (1 point). Assume that the radiant flux of your source is $\Phi_0 = 100\,W$, and that at a depth $d = 1.4\,m$, the detector records a current signal, which through the detector calibration indicates that the received flux at the detector is $\Phi(1.4\,m) = 62.9\,W$. Use this information to evaluate numerically the extinction coefficient that you found in part (a).

   c) (3 points). If the uncertainty in the depth measurement d is 1.5%, the uncertainty in your detector's measurement of the flux underwater is 2.2%, and the uncertainty in the radiant flux of your source is 3.1%, find a mathematical expression for the uncertainty in your estimate of the extinction coefficient, assuming the information given in part (b).

   d) (3 points). Suppose that the depth of the seafloor is $6.5\,m$. You now lower your detector to the seafloor below the light source, where the detector records a signal of $S(6.5m) = 0.813\,A$. Using this information and what you have determined or been given in the earlier parts of this problem, find an expression for the quantum efficiency of your detector at $\lambda = 0.4\;\mu m$ (the wavelength of the monocrhomatic lights source) and evaluate your expression numerically.

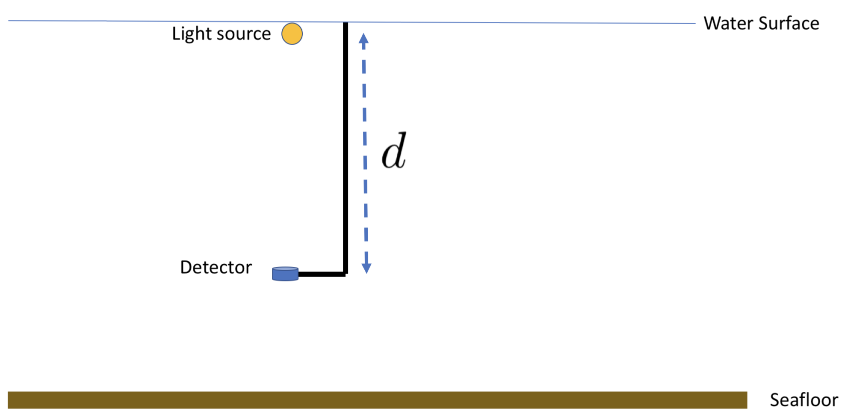


Figure for Problem 11: Light source just below the surface and detector suspended below the light with a set of rigid rods.

12. **Radiometry (10 Points).** You are conducting an experiment in a water tank in your laboratory. Initially you fill the tank with water from nearby coastal waters; the extinction coefficient of this water is $\beta\ [m^{-1}]$. After filling the tank, you place an isotropic light source at one end of the tank, and you suspend a screen across the middle of the tank blocking all light except for that light which passes through an embedded large Lambertian diffuser in the middle of the screen as shown in the Figure below. The light source has radiant intensity $I_s\ [W sr^{-1}]$ and is positioned at a distance x_s from the left end of the tank and at a depth h_s. You may assume that the light source is a point source, and you may also assume that there are no other sources of light in the set-up shown. The screen with the embedded diffuser is at a distance x_{diff} from the left end of the tank, and the diffuser is at a depth of h_{diff}. The diffuser has transmittance τ_{diff}. The camera is in close proximity to the diffuser, is located at a distance x_c from the left end of the tank, and faces the center of the large Lambertian diffuser at the same depth, h_{diff}, that is $h_c = h_{diff}$, as shown in the Figure below. The optics of the camera have transmittance τ_{cam}, and the camera has f-number $f^{\#}_{cam}$ with detector elements that are square with dimensions l_{pixel} on each side. The noise equivalent power of the camera detector elements is NEP_{cam}.

   a) (4 points). Find a mathematical expression for the minimum change in radiant intensity of the light source that could be detected in terms of one or more of the given variables.

   b) (2 points). Evaluate the minimum change in radiant intensity that you found in part a if $\tau_{diff} = 0.83$, $\beta = 0.17\ m^{-1}$, $x_{diff} = 2.75\ m$, $x_s = 1.1\ m$, $h_c = h_{diff} = 1.5\ m$, $h_s = 0.35\ m$, $x_c = 3.25\ m$, $\tau_{cam} = 0.89$, $f^{\#}_{cam} = 2.8$, $l_{pixel} = 20\ \mu m$, $NEP_{cam} = 1.45 \times 10^{-11} W$.

   c) (3 points). If the radiant intensity changes by a factor of m, what is the corresponding percentage change in the shot noise.

   d) (1 point). Evaluate your answer to part (c) numerically if $m = 1.1$.

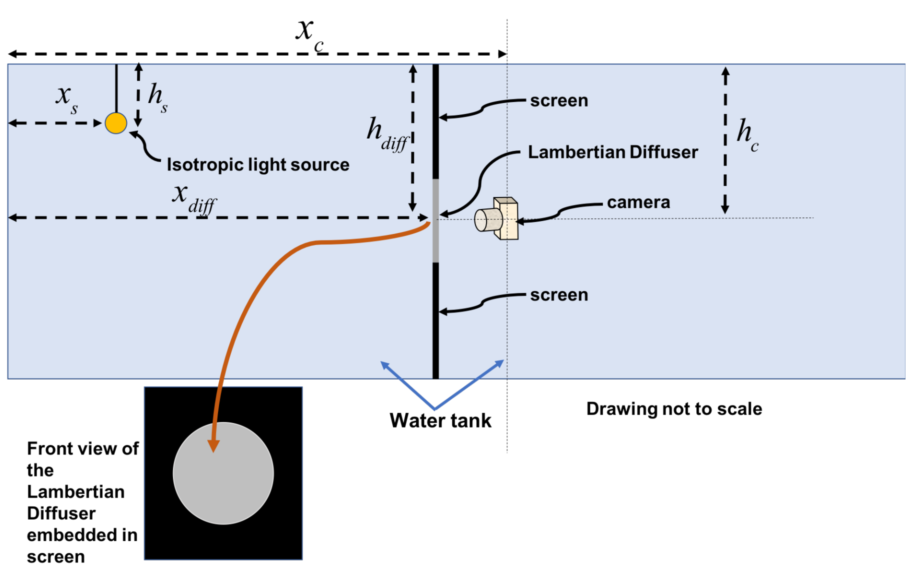


Figure for Problem 12: water tank with isotropic light source, screen with embedded Lambertian diffuser, and camera in close proximity to the diffuser.

Core 5: Image Processing and Computer Vision, Answer TWO Questions from Questions 13-15

13. **Image Processing and Computer Vision: Galaxy Morphology Classification (10 points).**

Table 1: Galaxy morphology dataset size.

Galaxy Type	Total Images	Training Set	Validation Set	Test Set
Elliptical	617	370	117	130
Spiral	513	308	97	108
Irregular	91	55	11	25

Astronomers classify galaxy morphology based on their visual appearance. The simplest scheme has three classes: spiral galaxies, elliptical galaxies, and irregular galaxies. You have decided to compete in a galaxy classification challenge with a $50,000 prize for the best results on the test set. The dataset has been pre-processed so that all galaxies are individually cropped. The numbers of images in the training, validation, and test sets are given in the Table.

a) (1 point) Explain why simple accuracy would be a bad metric for this dataset.

b) (1 point) Explain the typical purpose of the validation set and how to use it.

c) (2 points) You decide to use a convolutional neural network (CNN) to do this classification problem. What is the danger with using a deep CNN with 19 layers for this dataset? How might this danger be overcome?

d) (3 points) One property that may be beneficial to galaxy classification is rotation invariance; however, typical CNNs do not have this built into their architecture. Explain why this issue is usually ignored for largescale datasets, but might not be an issue that cannot be ignored here.

e) (3 points) Devise a method to make your CNN robust to changes in rotation in your CNN-based system. Give the steps to your algorithm and explain how it works.

14. **Image Processing and Computer Vision: Clustering and Segmentation (10 Points).**

This problem discusses clustering and using clustering to implement image segmentation algorithms.

Consider partitioning a set of T data points $D = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_T\}$, where $\mathbf{x}_t \in \mathbb{R}^d$.

a) (3 points) Assume that you are given an initial set of K clusters, i.e., $\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K$, where $\mathbf{c}_k \in \mathbb{R}^d$. Give pseudo-code for the standard k-means algorithm, i.e., Lloyd's method.

b) (1 point) What is the objective/loss/cost function to be minimized in k-means clustering? Assume Euclidean distance.

c) (2 point) Explain how k-means++ differs from k-means. What does it improve?

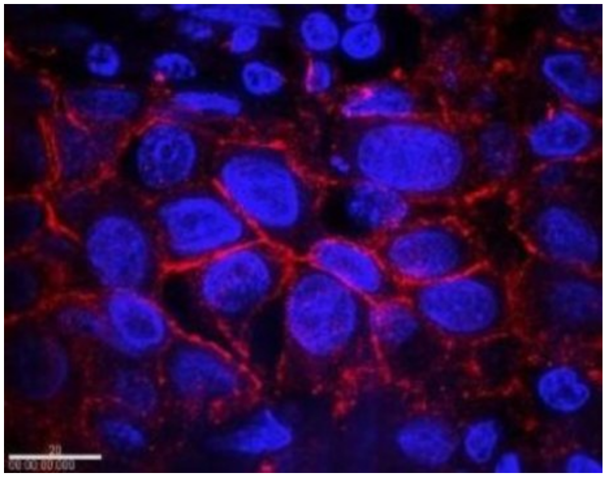
d) (1 point) If we initialize the k-means clustering algorithm with the same number of clusters but different starting positions for the centers, the algorithm will always converge to the same solution. Why or why not?

e) (1 point) You are using k-means clustering in some color space to segment an image. However, you notice that although pixels of similar color are indeed clustered together into the same clusters, there are many discontiguous regions because these pixels are often not directly next to each other. Describe a method to overcome this problem in the k-means framework.

f) (2 points) Compare the k-means and mean-shift clustering algorithms for segmentation, i.e., dimensionality, cluster shape, choosing the number of clusters, and robustness.

15. **Image Processing and Computer Vision: Image Processing and Binary Images (10 points).**

You have been hired to build a cell counting algorithm. To develop a system for this problem, you decide to use classic image processing techniques as a baseline. In this dataset, the cell nuclei are stained blue and the membranes are stained red.



a) (1 point) First, you need to convert your sRGB images to a monochrome representation. You need to preserve the information you need for cell counting, while suppressing information that is not useful. You recall HSV color space and decide to use Value to do this conversion, i.e., $V = \max(R, G, B)$. Explain why this was actually not the best idea and suggest an approach likely to work better.

b) (1 point) Could two monochrome images of identical sizes have identical histograms and can this happen if the images are not identical? If so, explain the general conditions in which this could occur. Assume the pixel value range is 0 to 255, and that there are 256 histogram bins centered at every possible integer.

c) (5 points) When transforming a monochrome image into a binary one using thresholding, we must choose a threshold. You decide to threshold the monochrome image using the Basic Global Thresholding Algorithm, also known as Balanced Histogram Thresholding method. Give pseudocode for this algorithm. The input should be a monochrome image I and the convergence criteria $v \geq 0$, which determines when to stop running the algorithm because it has converged. The output is the threshold value S.

   Initialize the starting threshold value T to the mean pixel value in I.

d) (3 points) Now that you have a good binary image, you need to use the connected components algorithm to identify all of the cells in the image. Explain how connected components works in English or give pseudocode.

Core 6: Graduate Laboratory: Complete All Parts

16. **Graduate Laboratory.** The project this year has been the design, development and testing of an intelligent baby monitor. These questions relate to the specific aspects of the project, and all must be answered.

    There are 4 basic components to the project:

    - Measurement of breathing rate
    - Measurement of the heart rate (pulse)
    - Measurement of the body temperature
    - Detection of baby noises and their classification (crying, etc.)

    Question 16, Part 1. For the breathing rate portion of the project:

    a) What device(s) were used to capture the signal from which the breathing rate was extracted?

    b) Describe in detail how the breathing rate was determined

    c) What limited the ability to extract a good measurement of breathing rate?

    Question 16, Part 2. For the heart rate (pulse):

    a) What device(s) were used to capture the signal from which the heart rate was extracted?

    b) Describe in detail how the heart rate was determined

    c) What limited the ability to extract a good measurement of heart rate?

    d) Which was easier to detect from the data, the breathing rate or the heart rate?

    Question 16, Part 3. For the body temperature:

    a) What device(s) were used to measure the body temperature?

    b) What limited the ability to extract a good measurement of body temperature?

    Question 4, Part 4. For the detection of baby noises:

    a) What device(s) were used to capture the baby noises?

    b) Describe in detail two ways in which the system could determine laughter, crying, and type of crying.

    c) For each of the 2 ways described in 4b, explain the advantages and disadvantages of each approach.