

R·I·T

2017 Imaging Science Ph.D. Qualifying Examination

June 9, 2017

9:00AM to 12:00PM

IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use AND STAY INSIDE THE BOX! Be sure to write your provided identification letter, the question number, and a sequential page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name on the supplied 5" x 8" paper containing your provided identification letter and place this in the small envelope, and then place this envelope along with your answer sheets in the large envelope.

**ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD
LIKE EVALUATED**

Identification Letter: _____

**THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE
PROCTOR UPON COMPLETION OF THE EXAM PERIOD**

Please show all work and write your answers clearly to obtain full credit for your solutions. Clearly label the problems that you are working out on a given page to ensure that the grader will be able to easily discern which problem is being worked on each page.

Core 1: Fourier Methods in Imaging, Select TWO from 1-3

1. Consider three imaging systems that act on one-dimensional functions.

(a) [20%] The transfer function of the first system is:

$$H_1[\xi] = STEP[\xi] \cdot \exp[+i \cdot \pi]$$

Find an expression for the image that results from this system if the input is $f[x] = \delta[x - 1]$.

(b) [10%] Sketch the image from part (a) as real part, imaginary part, magnitude, and phase

(c) [10%] The transfer function of the second system is:

$$H_2[\xi] = STEP[\xi] \cdot \exp[+i \cdot \pi \cdot \alpha_0 \cdot \xi \cdot STEP[\xi]]$$

What are the dimensions of the parameter α_0 ?

(d) [20%] Evaluate and sketch the image produced by this system for the same input used in part (a), again if the value of the parameter is $\alpha_0 = +2$.

(e) [20%] The transfer function of the third system is:

$$H_2[\xi] = \exp[+i \cdot \pi \cdot \alpha_0 \cdot |\xi|]$$

Sketch the real part, imaginary part, magnitude, and phase of this transfer function, again for the case $\alpha_0 = +2$.

(f) [20%] Evaluate and sketch the image produced by this system for the same input used in part (a).

2. A 1-D real-valued input object $f[x]$ has been “blurred” by convolution with the quadratic-phase impulse response:

$$h[x] = \exp \left[+i\pi \left(\frac{x}{\alpha_0} \right)^2 \right]$$

where α_0 is a positive real number with dimensions of length.

- (a) [40%] Find an expression for the impulse response $w[x]$ of the inverse filter.
(b) [60%] Determine the output:

$$f[x] * w[x] = g[x]$$

if $w[x]$ is the function from part (a) and:

$$f[x] = 4 \cdot \exp \left[+i\pi \left(\frac{x-4}{\alpha_0} \right)^2 \right] + 2 \cdot \exp \left[+i\pi \left(\frac{x+2}{\alpha_0} \right)^2 \right] + \exp \left[+i\pi \left(\frac{x+3}{4\alpha_0} \right)^2 \right]$$

and explain the features of this result.

3. Consider a camera with a discrete sensor that is an $N \times N$ array of square pixels of width d_0 spaced by the the same pitch in the orthogonal directions: i.e., that $\Delta x = \Delta y \geq d_0$. This camera is used to image a band-limited object.
- (a) [10%] Specify the limiting frequency of the image on the sensor that ensures that there is no aliasing; this is the Nyquist frequency.
 - (b) [20%] Assuming that the input object is bandlimited as determined by part (a), find an expression for the 1-D profile of the normalized MTF due to the finite sensor size along the x -axis if the pixel dimension d_0 is smaller than the pixel pitch Δx . In other words, the fractional area of the discrete sensor array that is sensitive to light (i.e., the “fill factor” of the array) is less than 100%.
 - (c) [10%] Evaluate the numerical value of the MTF at the Nyquist sampling frequency.
 - (d) [10%] Graph the 1-D MTF evaluated in part (b) in the case where the pixel size is equal to the pixel pitch: $d_0 = \Delta x$; the fill factor of this array is 100%.
 - (e) [20%] Now consider the case of a sensor with 100% fill factor that may be translated relative to the image of a stationary object projected by the optics onto the sensor plane. The following procedure is used to collect the image: (1) the image is recorded with the sensor in its original position; (2) the sensor is translated along the x -direction by $\frac{\Delta x}{2}$ and another image recorded; (3) the sensor is translated along the y -direction by $\frac{\Delta x}{2}$ and another image recorded; and (4) the sensor is translated along the x -direction by $-\frac{\Delta x}{2}$ and a fourth image recorded. The final image is formed by combining these four images to create a $2N \times 2N$ array of pixels with a sample pitch of $(\Delta x)_1 = \frac{\Delta x}{2}$. Determine the Nyquist frequency of the new imaging system and the value of the MTF of the system at this new Nyquist frequency.
 - (f) [10%] Graph the 1-D MTF of the new system.
 - (g) [20%] Characterize the possible benefits and limitations of images of stationary objects created with the new system.

Core 2: Optics, Select TWO from 4-6

Each answer must be accompanied by supporting illustrations. Be sure to label the axes and all important characteristics in your sketch. Illegible work will not be graded. Partial credit will be awarded if it demonstrates a level of competency.

Please use a straight edge ruler and a single-purpose calculator when needed. If you not have these, please tell the proctor, who will distribute them.

4. (Diffraction) Two point sources of wavelength λ and equal strength are located in the object plane at the distances $x_0 = \pm a$ from the optical axis. Assume the sources are mutually coherent (i.e., light from the sources can interfere).
- (a) Using Huygen's principle, determine a complex algebraic expression (in terms of x, z, x_0 , and any other pertinent variables) for the net electric field at the point x in a plane located a distance z from the object. Simplify the expression by use of a series expansion, assuming $x \pm a \ll z$.
 - (b) A lens of focal length \mathbf{f} and semidiameter R is placed a distance $z = 2\mathbf{f}$ from the point sources. Write an expression for the electric field at the immediate output face of the lens.
 - (c) Using the Huygens-Fresnel integral for the field in the x' plane, a distance Δz from the initial field $E[x]$:

$$E[x', \Delta z] = \frac{\exp[+ik \cdot \Delta z]}{i\lambda \cdot \Delta z} \int_{-\infty}^{+\infty} E[x] \exp\left[+ik \cdot \frac{(x - x')^2}{2 \cdot \Delta z}\right] dx$$

Determine a simplified expression for the complex electric field if $\Delta z = 2\mathbf{f}$. (You must evaluate the integral to receive credit for this problem.)

- (d) On the diagrams that you have drawn for the above problems, sketch the rays that provide a geometrical optics solution. Based on this sketch, write a simple expression for the electric field in the geometrical optics approximation. Prove that this expression agrees with your solution in part (c) in the limit as $R \rightarrow \infty$.

5. Geometrical Optics: An optical relay is a periodic series of lenses that transfers rays from one end of a device to another.
- (a) Using ray tracing, prove that a series of identical lenses of focal length \mathbf{f} , separated by a distance $d = \mathbf{f}$, will transfer rays without magnification if the object is placed a distance $d_0 = \mathbf{f}$ from the first lens. (Draw the initial object large enough to avoid drawing errors – roughly 2 cm should be sufficient).
 - (b) How many lenses are required to obtain an erect image of magnification $M = +1$? Support your answer with your ray diagram. (An image will appear a distance $d_i = \mathbf{f}$ after the last lens in the sequence.)
 - (c) Determine the magnification of this relay system if the object is instead placed a distance $d_o = 2\mathbf{f}$ from the initial lens in the sequence. (Hint: this may be solved using similar triangles. You may wish to make a ray tracing drawing to check your solution.)
 - (d) Verify your result in part (b) using ABCD matrices. That is, prove that the image height is equal to the object height if $d_o = d_i = \mathbf{f}$. (Hint: you may wish multiply the matrices in an organized fashion, e.g., pair-wise).

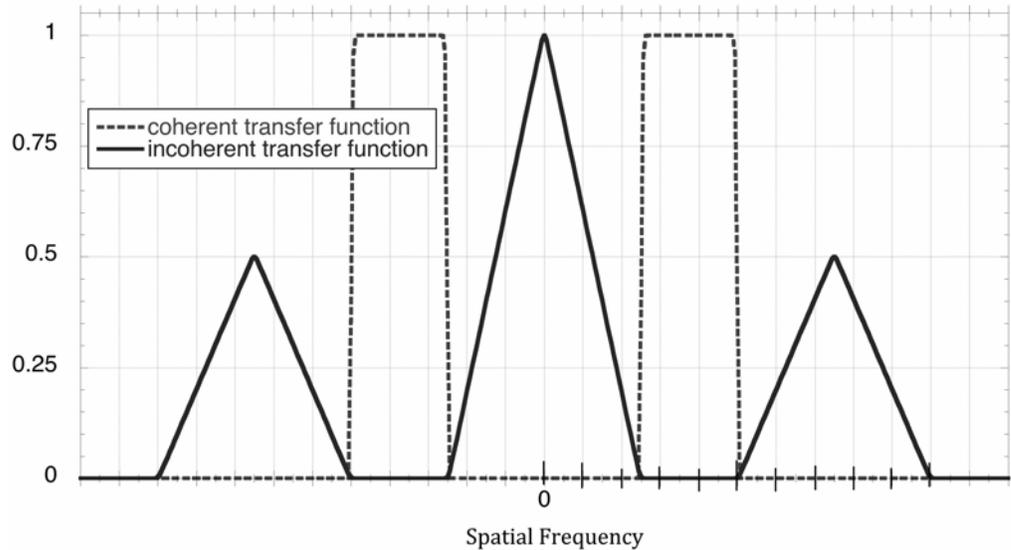
6. Modulation transfer function: The coherent and incoherent transfer functions for a one-dimensional lens of focal length $f = 158$ mm with a central obscuration and outer width R_{out} are shown in the plot below for an image distance of $d_i = 2f$. The OTF is a real-valued function. The incoherent cutoff frequency is $500 \frac{\text{cycles}}{\text{mm}}$ and the wavelength is $\lambda_0 = 633$ nm.

- (a) The aperture is comprised of two rectangle functions. Using the plot below and the equation for the incoherent cutoff frequency, determine the values of the inner and outer sizes R_{in} and R_{out} of the lens aperture function in units of mm.
- (b) An object having an intensity profile given by

$$I_o [x] = 1 + \cos \left[2\pi \frac{x_o}{\Lambda} \right]$$

is placed in the object plane, where $\Lambda = 2.667 \mu\text{m}$. Determine the intensity profile of the image $I_i [x_i]$, using the incoherent transfer function presented below.

- (c) Compare the obscured lens in the previous parts to an unobscured lens having the same outer width R_{out} and the same incoherent cutoff frequency. That is, plot the incoherent transfer function of the unobscured aperture. Determine the intensity profile of the image $I_i [x_i]$ for the unobscured lens assuming the same object given in (b).
- (d) Prove that the image contrast obtained for the obscured lens is twice as large as that for the unobscured lens.



Core 3: Vision, Select TWO from 7-9

7. The human visual system has been optimized over a long period of evolution. Today, the ‘typical eye’ has an acuity of 20/20 under ‘optimal conditions,’ has a field of view extending over 180° horizontally, can detect and differentiate among a huge number of different spectral mixtures while being relatively insensitive to changes in the spectral content of the illuminant, and over a range of illumination that spans at least 10 orders of magnitude, without saturating or suffering from noticeable noise.

But physical measurements of the optics of the human eye indicate (i) significant chromatic aberration when broadband illumination is used, (ii) significant spherical aberration even when illumination is limited to a narrow range of wavelengths, and (iii) severe loss of image quality even a few degrees away from the optical axis.

- (a) [20%] Describe how it is possible for humans to have such high perceived visual image quality when we know from physical measurements that severe chromatic and spherical aberrations are inherent in the relatively simple, two-element optical design of the eye.
- (b) [20%] Describe how it is possible for humans to have 20/20 acuity and such a wide field of view given what we know about how acuity falls off so dramatically off axis. [Include in your answer a definition of “20/20 acuity” and how it is measured.]
- (c) [20%] How does the visual system achieve such a high *between-scene* dynamic range? In other words, how is it possible to perceive detail in bright sunlit scenes at one time, then under moonlit and cloudy conditions later?
- (d) [20%] How does the visual system achieve such a high *within-scene* dynamic range? In other words, how is it possible to simultaneously perceive detail in bright, sunlit regions and in dark shadows?
- (e) [20%] We know from physiological studies that ‘color’ information is carried in the human visual system in only three neural channels, yet most observers can differentiate among millions of different spectral combinations. On the other hand, it is also true that there are sets of spectral ‘metameric pairs’ that appear identical, despite the fact that their spectra are completely different. Discuss this apparent dichotomy, including in your discussion some reason(s) that humans evolved to have such a small number of chromatic channels.

8. When trying to understand the performance of the human visual system (HVS), vision scientists measure both the modulation-transfer function (MTF) and contrast-sensitivity function (CSF) of observers.

- (a) [30%] Describe the MTF. Include in your description a sketch of the MTF for a typical observer under ideal conditions, with both axes labeled and scaled. Discuss why the MTF isn't "perfect," i.e., explain why the curve varies from an ideal system that passes all spatial frequencies perfectly without affecting the output.
- (b) [30%] Describe the CSF. Include in your description a sketch of the CSF for a typical observer under ideal conditions, with both axes labeled and scaled. Discuss why the CSF isn't "perfect," i.e., explain why the curve varies from an ideal system that passes all spatial frequencies perfectly without affecting the output.

NASA and DARPA are collaborating on a humanoid robot for use in extraterrestrial exploration

- (c) [20%] Among the sensors now available in robots are *binocular-vision sensors*. If you were to have an analogous metric to the MTF of the HVS, what would you measure in the robot's "visual system?"
- (d) [20%] If you were to have a measure parallel to the CSF of the HVS, what would you measure in the robot's "visual system?"



Jeff Pelz cannot be serious including this image of the illegitimate offspring of Gort and Robbie the Robot in this test.

9. This is Figure 6 from Daly's classic paper, *Visible differences predictor – an algorithm for the assessment of image fidelity*, which introduced an explicit 2D model for observer CSF. In referring to the figure, Daly says, “Shown in Figure 6 is a typical CSF for the 2-D frequency plane, showing the bandpass nature and oblique effect.”

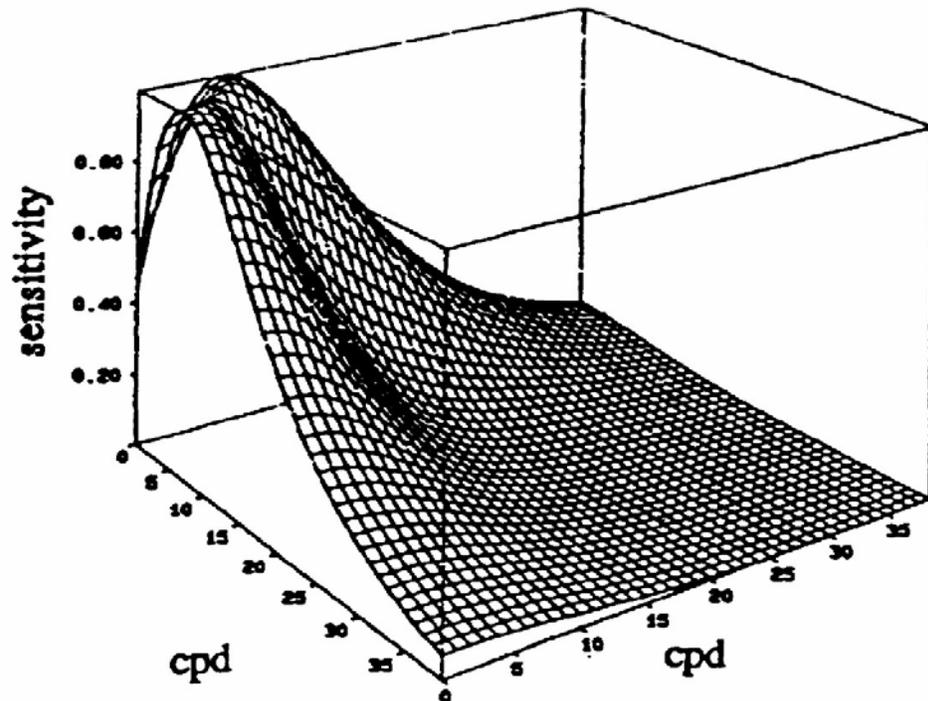


Figure 6

- (a) [20%] Describe the “bandpass nature” Daly refers to, including the spatial frequency of peak response and f_{\max} along the horizontal and vertical axes.
- (b) [30%] Explain the reason for this bandpass nature; specifically, the reason(s) for the loss of response at high and low frequencies.
- (c) [30%] What is the ‘oblique effect?’
- (d) [20%] Discuss to what extent man-made optical systems share either of these characteristics.