

R·I·T

2018 Imaging Science Ph.D. Qualifying Examination

June 1, 2018

9:00AM to 12:00PM

IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use AND STAY INSIDE THE BOX! Be sure to write your provided identification letter, the question number, and a sequential page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name on the supplied 5" x 8" paper containing your provided identification letter and place this in the small envelope, and then place this envelope along with your answer sheets in the large envelope.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD
LIKE EVALUATED

Identification Letter: _____

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE
PROCTOR UPON COMPLETION OF THE EXAM PERIOD

Core 1: Fourier Methods in Imaging. Answer TWO Questions from Questions 1-3

1. **Fourier Methods in Imaging (10 points).** Consider an imaging system that acts on 2-D spatial functions with quadratic-phase impulse response:

$$h[x, y; \lambda_0, z_1] = \exp \left[+i\pi \frac{x^2 + y^2}{\lambda_0 z_1} \right]$$

where λ_0 and z_1 are constant parameters each with dimensions of length. The impulse response has the form of an expanding paraboloid.

- (a) Evaluate the impulse response $w[x, y; \lambda_0, z_1]$ and the transfer function $W[\xi, \eta; \lambda_0, z_1]$ of the inverse filter for this system.
- (b) In the case where the input to the system is a converging paraboloid:

$$f_1[x, y] = \exp \left[-i\pi \frac{x^2 + y^2}{\lambda_0 z_1} \right]$$

Evaluate the output amplitude $g_1[x, y; \lambda_0, z_1]$ and squared magnitude $|g_1[x, y; \lambda_0, z_1]|^2$ from the system.

- (c) Evaluate the output amplitude $g_2[x, y; \lambda_0, z_1]$ and squared magnitude $|g_2[x, y; \lambda_0, z_1]|^2$ of the system if the input function is $f_1[x, y]$ truncated by a “square” aperture with sides of width b_0 .

2. **Fourier Methods in Imaging (10 points).**

Consider an input function $f[x]$ where $0 \leq f \leq 1$. A nonlinear operator \mathcal{N} is applied to this function:

$$\begin{aligned} \mathcal{N}\{f[x]\} &= g[x] \\ &\equiv (1 - GAUS[f[x]]) \cdot RECT\left[f - \frac{1}{2}\right] \\ &= \left(1 - \exp\left[-\pi(f[x])^2\right]\right) \cdot RECT\left[f - \frac{1}{2}\right] \end{aligned}$$

Examples of outputs are shown in the table:

$f[x]$	$g[x]$
2	$\left(1 - \exp\left[-\pi(2)^2\right]\right) \cdot RECT\left[2 - \frac{1}{2}\right] = 0$
1	$1 - GAUS[1] = 1 - e^{-\pi} \cong 0.957$
$\frac{1}{2}$	$1 - GAUS\left[\frac{1}{2}\right] = 1 - e^{-\frac{\pi}{2^2}} \cong 0.544$
0	$1 - GAUS[0] = 0$
-1	$\left(1 - \exp\left[-\pi(-1)^2\right]\right) \cdot RECT\left[-1 - \frac{1}{2}\right] = 0$

- (a) Sketch $g[x]$ if the input function $f[x] = TRI[x]$.
- (b) Evaluate $G[\xi]$ if the input is the biased cosine $f[x] = \frac{1}{2} + \frac{1}{2} \cos[2\pi x]$, which means that $0 \leq f \leq 1$. You may make reasonable approximations, but state what you use.

3. Fourier Methods in Imaging (10 points).

Consider the 2-D function:

$$f[x, y] = \delta[x] \cdot 1[y] + 1[x] \cdot \delta[y]$$

- (a) (10%) Sketch $f[x, y]$ in some manner that shows its 2-D form.
- (b) (25%) Evaluate and sketch $F[\xi, \eta] = \mathcal{F}_2\{f[x, y]\}$
- (c) (10%) Sketch the function $f[x, y]$ after rotation about the origin by θ radians to create $R_\theta\{f[x, y]\} \equiv r[x, y; \theta]$.
- (d) (25%) Describe the difference between the 2-D spectra of $f[x, y]$ and $r[x, y; \theta]$, i.e., between $F[\xi, \eta]$ and $R[\xi, \eta; \theta]$.
- (e) (30%) The spatial integral of the set of rotated replicas of $f[x, y]$ is:

$$\int_{\theta=-\frac{\pi}{2}}^{\theta=+\frac{\pi}{2}} r[x, y; \theta] d\theta = \frac{1}{\sqrt{x^2 + y^2}}$$

Evaluate the spectrum $\mathcal{F}_2\left\{\frac{1}{\sqrt{x^2+y^2}}\right\}$ (HINT: Hankel transform)

Core 2: Optics. Answer TWO Questions from Questions 4-6

Optics Instructions: Each answer must be accompanied by supporting illustrations. Be sure to label the axes and all important characteristics in your sketch. Illegible work will not be graded. Partial credit will be awarded if it demonstrates a level of competency. Please use a straight edge ruler (or paper edge) and a single purpose calculator when needed. If you do not have one, please tell the proctor.

4. **Optics (10 points). Fraunhofer Diffraction.** The moon is $Z_m = 3.8 \times 10^8 m$ from the earth. You wish to direct a beam of wavelength $\lambda = 1\mu m$ from the earth to the moon, so that an Airy disk of radial size, R_m illuminates a small portion of the lunar surface.
- State how this may be achieved using only Fraunhofer diffraction (without a lens). You must provide a detailed sketch of the system. Also state any assumptions about Fraunhofer diffraction.
 - Express the electric field at Z_m in terms of an integral over the electric field of the beam on the Earth. You MAY NOT use shorthand notation such as $FT[E(x)]$. Evaluate the integral by hand (you must show your work) and determine expressions for the characteristic angular beam width $\Delta\theta$ and Airy disk size R_m at the distance Z_m .
 - If you design your Earth system to achieve $R_m = 20m$, will the far-field condition be satisfied? You must prove your answer (i.e., evaluate the diffraction length and compare it to Z_m).
5. **Optics (10 points). Imaging with the Eye.** When gazing at an object at the Near Point, the focal length of an eye is found to be $f_{np} = 1.85cm$.
- An object of height $h = 50\mu m$ is placed at the Near Point of the eye, $Z_{np} = 25cm$. Determine the image height on the retina (expressed in μm). You must provide an instructive ray diagram to receive credit.
 - Determine the incoherent cut-off frequency for part (a), assuming a pupil diameter $D_{pupil} = 4mm$ and a wavelength $\lambda = 0.5\mu m$. Express your answer in units of oscillations per mm .
 - A magnifying lens of focal length $f_{ml} = 12cm$ is placed between the object and the eye, with the object located $11.5cm$ from the lens. The focal length of the eye for a far point is $f_{fp} = 2.0cm$. Assume the distance between the magnifying lens and the front of the eye is $1.0cm$. Determine the image height on the retina. You must provide an instructive ray diagram to receive credit.
6. **Optics (10 points). Paraxial Ray Tracing.** For each of the following, provide a detailed sketch, along with all calculations and derivations. To receive credit, YOU MUST PROVE each answer (not simply write a memorized result). Use the convention $y_2 = Ay_1 + B\theta_1$, $\theta_2 = Cy_1 + D\theta_1$.
- Derive the $ABCD$ matrix for ray propagation over a distance d through a homogeneous isotropic medium.
 - Derive the $ABCD$ matrix for ray propagation across an interface between the first material having an index of refractive, n_1 , and the second material having an index of refractive, n_2 .
 - A Galilean telescope is comprised of a positive and negative lens of respective focal lengths $f_1 = f$ and $f_2 = -f/10$. The $ABCD$ matrix for a thin lens of focal length f_0 has elements $A = D = 1$, $B = 0$, and $C = -1/f_0$. Determine the system (lens-space-lens) $ABCD$ matrix for the telescope as a function of the lens separation distance, d . Use your result to determine an expression for d (in terms of f) that produces a collimated output beam when the incoming beam is also collimated. State whether your answer is reasonable.

Core 3: Human Vision. Answer TWO Questions from Questions 7-9

7. **Human Vision (10 Points).** Your team is working on a new head-mounted display (HMD) to be used by neurosurgeons to perform tele-operations on patients in operating rooms a few thousand km away. The display is to be binocular, driven by two high-resolution cameras on an operating robot controlled by the neurosurgeon.

Describe the important parameters driven by the human visual system that you would consider in designing the HMD. Include in your description the parameter, the reason it is important, relevant values, and any considerations that should be taken into account. For example (briefly): Parameter 1 - color, which is important because color carries information in the context of an operation for describing tissue types, presence of blood, etc.. Relevant values - 3 color channels to drive the S, M, & L wavelength cone classes; $\lambda \approx 400 - 700 \text{ nm}$ to cover the visible range. Considerations - some people are color blind or color-anomalous, i.e., missing one or more cone classes or the spectral response of one or more cone classes is shifted.

8. **Human Vision (10 Points).** The human visual system works over a huge range of illumination conditions. Describe the range and the mechanisms that allow this flexibility, including any trade-offs that are made to permit the large range.

The human visual system also works over a large range of object distances. Describe the range and the mechanisms that allow this flexibility.

9. **Human Vision (10 Points).** A helicopter flies overhead close enough that you can see the pilot's face clearly. At that distance, you see that the helicopter is painted with a fine pattern of black and white lines, so you take out your custom CCD video camera and start recording video of the helicopter as it flies away.

When you play the video back on a display with the same spatial and temporal resolution as the CCD sensor (i.e. it shows every pixel of every frame captured by the camera) you notice two interesting things. First, the main rotor appears to be stationary, not rotating. Second, while the pattern of black and white lines is clearly visible at the beginning of the video, as it flies away from your location, the contrast of the lines first fades out towards a gray, then reappears, but in an uneven, wavering pattern of much wider light and dark lines, then fades back to gray again. When you were looking directly at the helicopter the blades were clearly rotating, and while the lines eventually faded to a uniform gray, you never saw the wider line pattern before they faded to gray.

i. What caused the two phenomena described above? Explain the origin of both effects, and describe what (if anything) you could determine about the characteristics of the helicopter given the technical specifications for your video camera.

ii. Why didn't you experience the same phenomena when you were viewing the helicopter directly?