R·I·T

2015 Imaging Science Ph.D. Qualifying Examination
June 15, 2015
9:00AM to 12:00PM

IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use AND STAY INSIDE THE BOX! Be sure to write your provided identification letter (located below), the question number, and the page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name on the supplied 5" x 8" note card containing your provided identification letter and place this in the small envelope, and then place this envelope along with your answer sheets in the large envelope.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: ________________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD

R·I·T

2015 Imaging Science Ph.D. Qualifying Examination
June 15, 2015
1:00PM to 3:00PM

IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you useAND STAY INSIDE THE BOX! Be sure to write the identification letter provided to you this morning, the question number, and the page number for each answer in the upper right-hand corner of each sheet of paper that you use.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: ________________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD

1. **Optics for Imaging (10 pts)**

In 1953 Frits Zernike was awarded the Physics Nobel prize for the invention of a 4-f phase contrast imaging system, whereby a transparent object having an electric field profile, $E_o(x)$ in the x-plane, can be made visible by use of a phase mask $t(x')$ in the x'-plane. The coherently illuminated object is placed in the front focal plane of lens-1. The mask is placed in the back focal plane of lens-1, which is also the front focal plane of lens-2. Assume the lenses have the same focal lengths, $f_1 = f_2$.

(a) For the object field below, $E_o(x)$, explicitly determine (by use of an integral) the field at the input face of the phase mask, $E'(x')$. Do not use short-hand notation. You must provide an explicit integral and define all variables.

$$E_o(x) = \exp[i\beta(x)]$$

where $\beta(x) = \varepsilon cos(2\pi x/\Lambda)$ and $\varepsilon \ll 1$.

Assume the paraxial approximation (e.g., the wavelength $\lambda \ll \Lambda$). HINT: Justify why you may use only the first two terms of the Taylor series expansion of $E_o(x)$.

(b) Determine the field, $E_i(x'')$ in the image plane (x''-plane), assuming

$$t(x') = \begin{cases} \exp(i\pi/2), & |x'| < s \\ 1, & |x'| \geq s \end{cases}$$

where $s = \lambda f/\Lambda$.

(c) Prove that the intensity contrast (of the image) afforded by the mask is approximately 2ε.

2. **Optics for Imaging (10 pts)**

A one-dimensional aperture of diameter $2a$ is placed in the plane $z = 0$ and is uniformly illuminated with coherent light.

(a) Use the angular spectrum method to determine an integral expression for the electric field at an arbitrary distance z. Provide your solution in the form of a single integral:

$$E(x, z) = \int_{-\infty}^{\infty} g(k_x) \exp(ik_x x) dk_x$$

where $g(k_x)$ is a non-integral analytic expression. Assume paraxial waves.

(b) Plot the angular spectrum along the kx axis and use the plot to discuss the different conditions under which paraxial or non-paraxial waves may be assumed. HINT: use the first zero of the angular spectrum to characterize the width of the distribution.

(c) Describe the waves having $|k_x| > (2\pi/\lambda)$. What can you say about their propagation through space?

3. **Optics for Imaging (10 pts)**

The incoherent optical transfer function of a one-dimensional lens of diameter $D = 2a$ and focal length f is shown below. Assume the optical configuration provides unity magnification.

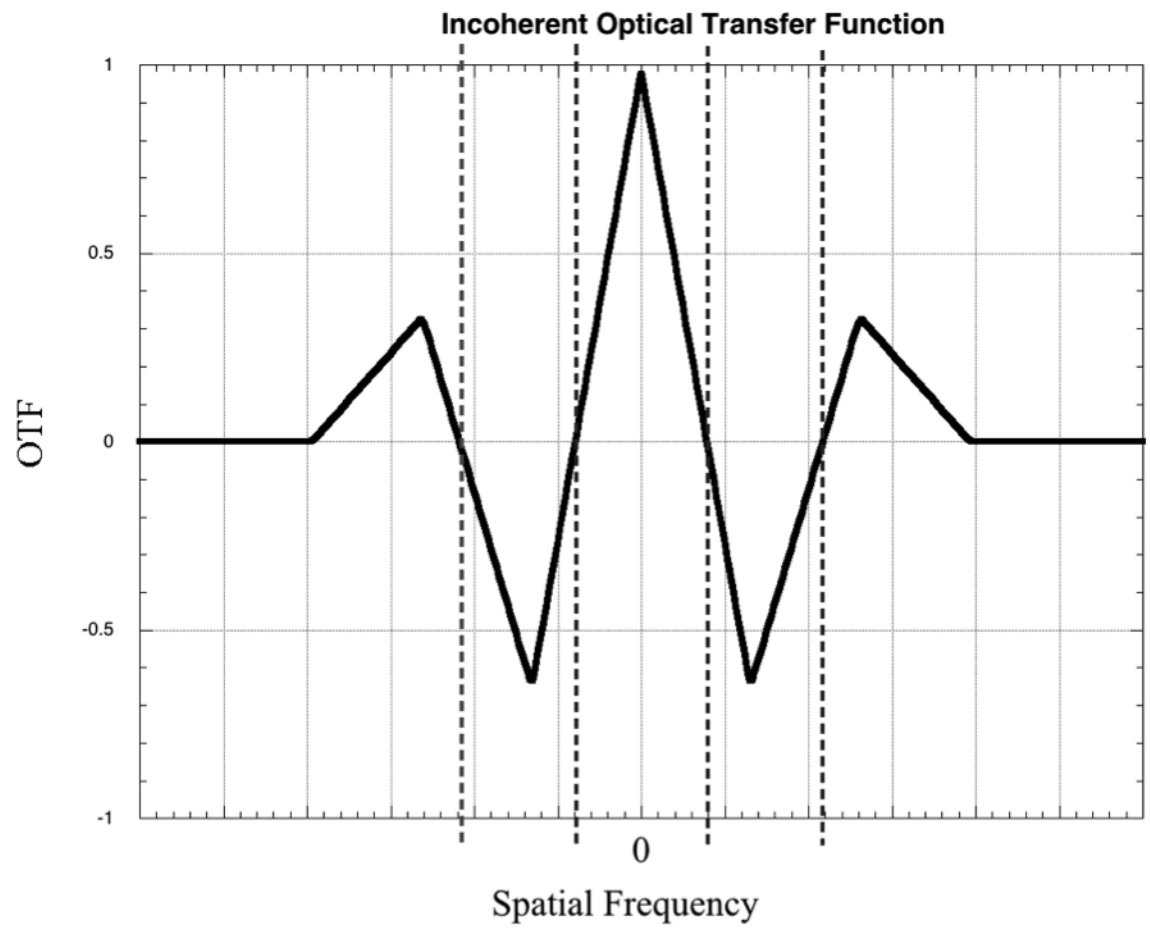
(a) Express the cut-off frequency, f_{cut} (beyond which the OTF is zero valued) in terms of the diameter D, wavelength λ, and focal length, f.

(b) An object has an intensity profile given by the equation below. Determine the image profile.

$$I_o(x) = A_0 + A_1 \cos(2\pi f_1 x) + A_2 \cos(2\pi f_2 x) + A_3 \cos(2\pi f_3 x)$$

where $f_1 = 0.2f_{cut}$, $f_2 = 0.3f_{cut}$, and $f_3 = 1.40f_{cut}$.

(c) Determine the contrast of the image.



4. **Human Visual System (10 pts)**

   Snellen acuity is a measure of spatial acuity. In backwards nations (such as the United States...), ratios such as '20/20' are used to indicate average corrected vision, and the value '20/200' is used as the threshold to define 'legally blind' in many jurisdictions.

   (a) What do the ratios (e.g. '20/20' and '6/60') signify?

   (b) Are there fundamental limits on the 'low' end of Snellen acuity? For example, could an observer have '20/2000' acuity? Defend your answer. If your answer is 'yes,' estimate that limit and explain the reason(s) for the limit.

   (c) Are there fundamental limits on the 'high' end of Snellen acuity? For example, could an observer have '20/2' acuity? Defend your answer. If your answer is 'yes,' estimate that limit and explain the reason(s) for the limit.

5. **Human Visual System (10 pts)**

   The human Contrast Sensitivity Function (CSF) is often described as 'analogous' to the Modulation Transfer Function (MTF) of optical systems.

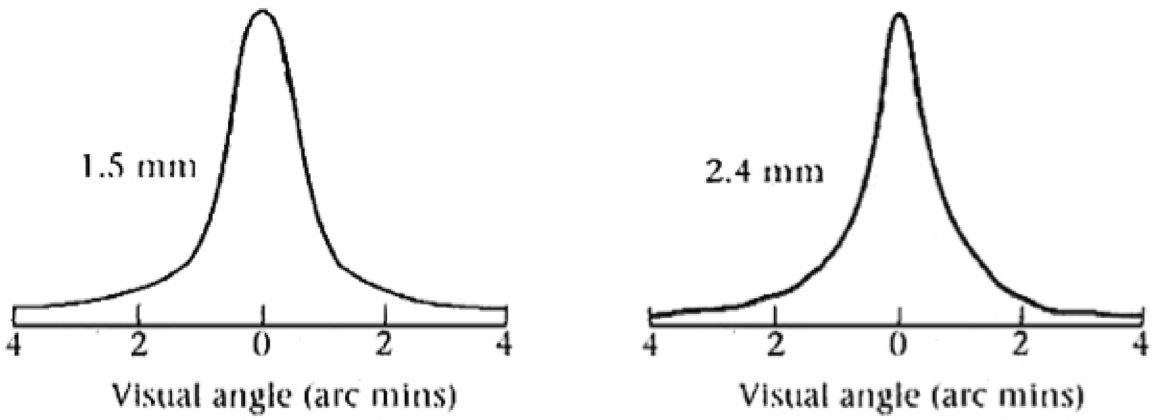
   (a) Discuss the CSF of an average human observer under ideal conditions and broad-band illumination, and explain how you could measure it. Include a sketch of the CSF plot, with the axes labeled and scaled.

   (b) Describe at least one way that the human CSF is similar to the MTF of an optical system, and at least one way that it differs.

   (c) The luminance CSF (i.e., for patterns that vary from black to white) is very different than a chrominance CSF measured (for example) with patterns that vary between yellow and blue. Describe the important differences between the luminance and chrominance CSFs (sketching the two plots if you would like), and explain why they must be different, based on the structure of the human visual system.

6. **Human Visual System (10 pts)**

The figure below shows optical line-spread functions (LSFs) of the human eye measured in the 1960s. They are the result of double pass measurements in which a narrow line was projected through the cornea and crystalline lens onto the retina (1st pass), then photographed through the cornea and crystalline lens (2nd pass). The published result was calculated by compensating for the double-pass through the optical system. The LSF for two pupil diameters (1.5 and 2.4 mm) are shown below.

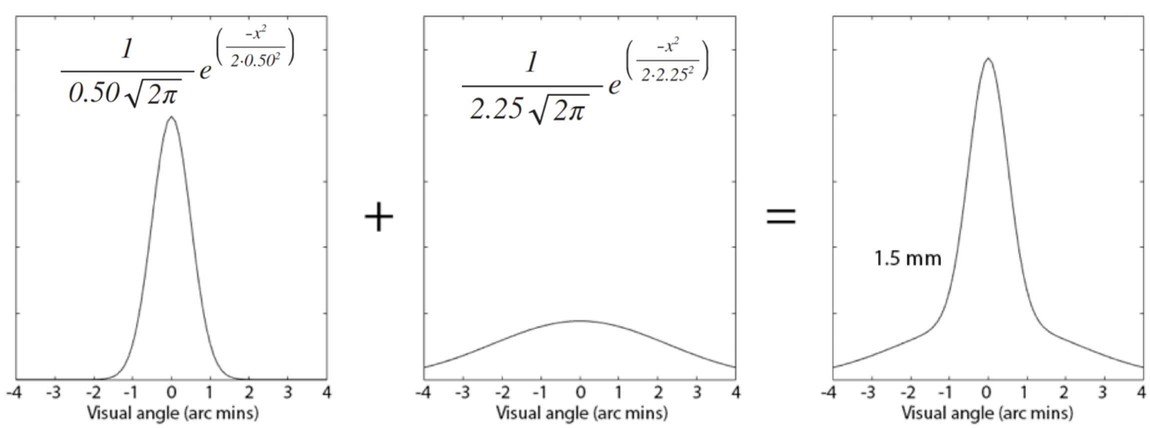


F.W. Campbell and R.W. Gubisch. Optical quality of the human eye. Journal of Physiology, 186:558--578, 1966

These line-spread functions are well modeled as the sum of two Gaussian distributions. For example, the **LSF of a 1.5mm pupil eye** is modeled as the sum (where x represents visual angle in arc minutes):

$$\frac{1}{0.50\sqrt{2\pi}}\,e^{\left(\frac{-x^2}{2\cdot 0.50^2}\right)}+\frac{1}{2.25\sqrt{2\pi}}\,e^{\left(\frac{-x^2}{2\cdot 2.25^2}\right)}$$

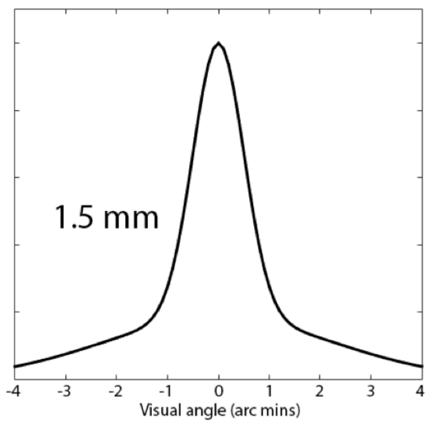
Graphically, for the 1.5 mm pupil:



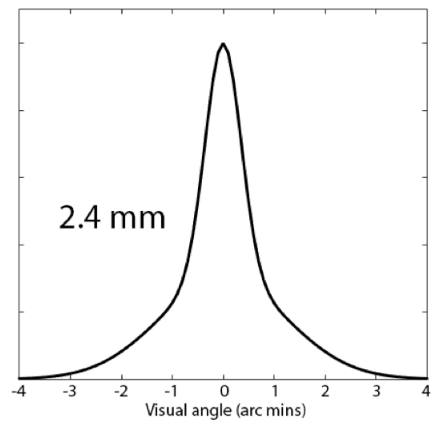
Similarly, the **LSF of a 2.4mm pupil eye** is modeled as the sum:

$$\frac{0.4}{0.35\sqrt{2\pi}}e^{\left(\frac{-x^2}{2\cdot 0.35^2}\right)}+\frac{0.6}{1.25\sqrt{2\pi}}e^{\left(\frac{-x^2}{2\cdot 1.25^2}\right)}$$

The two models are shown below:



$$\frac{1}{0.50\sqrt{2\pi}}e^{\left(\frac{-x^2}{2\cdot 0.50^2}\right)}+\frac{1}{2.25\sqrt{2\pi}}e^{\left(\frac{-x^2}{2\cdot 2.25^2}\right)}$$

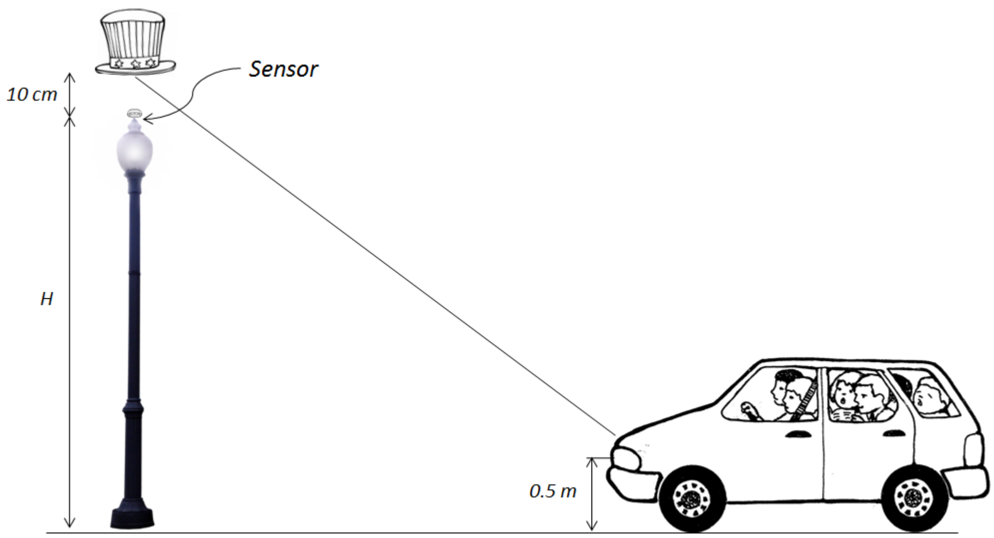


$$\frac{0.4}{0.35\sqrt{2\pi}}e^{\left(\frac{-x^2}{2\cdot 0.35^2}\right)}+\frac{0.6}{1.25\sqrt{2\pi}}e^{\left(\frac{-x^2}{2\cdot 1.25^2}\right)}$$

Estimate the MTF of the optical system (the eye) with 1.5mm and 2.4mm pupils. Show your answer as an MTF plot with labeled and scaled axes.

7. **Radiometry (10 pts)**

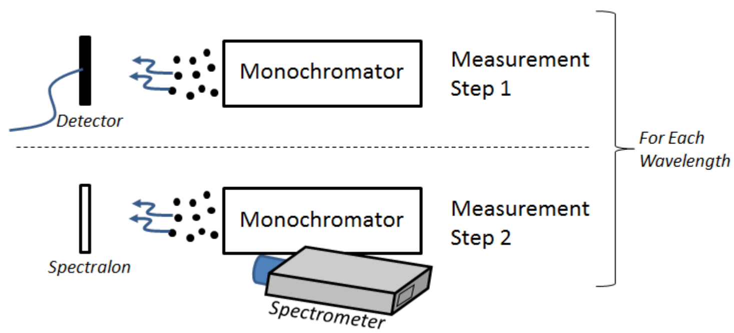
The city of Rochester is placing fourth of July holiday hats, of diameter 20 cm, directly above (i.e., at a distance of 10cm) city street lights (of height $H = 5\,m$) and is concerned that car headlights (with typical intensity of 10 watts per steradian) reflected from the *underside* of the decoration may trigger the photo diode detector, of responsivity 0.234 amps per watt and area of $1\,cm^2$, causing the street lights to be switched off by passing cars. A misaligned headlight is noted to have an elevation angle of 20 degrees while the detector has a noise rate of 10,000 electrons per $10\,\mu s$. If the typical air extinction is $10 \times 10^{-3}\,m^{-1}$ and the light is known to turn off with an SNR $\geq$ 300, what is the maximum reflectance the decorations are allowed to be? NOTE: assume the underside of the hat is uniform all the way across with no hole and the *side* of the hat does not play any role in the problem.



Possibly Useful Information
$A = 2898\ [\mu m\ K]$
$\sigma = 5.67 \times 10^{-8}\ [W\ m^{-2}\ K^{-4}]$
$\mathrm{h} = 6.624 \times 10^{-34}\ [J\ s]$
$\mathrm{c} = 3.0 \times 10^{8}\ [m\ s^{-1}]$
$\mathrm{e} = 1.60217 \times 10^{-19}$ [coulombs]

8. **Radiometry (10 pts)**

PART A) (8 pts.) I'm trying to compute the total responsivity of a $1cm^2$ silicon photo detector from collected data at a couple wavelengths (*i.e.*, 500 and 510 nm). I place my detector in front of a monochromator where light falls incident onto the detector. In a second measurement, a spectrometer produces readings of 15 and 12 $[Wm^{-2}sr^{-1}nm^{-1}]$, for 500 nm and 510 nm light, respectively, from a piece of Spectralon® at the same location and size as the detector. My detector is connected to a readout device where the output electron rate of the detector is known to vary as $\frac{N_{ele}}{t} = G\lambda_{nm} + b$, where λ_{nm} is the wavelength in nanometers, $G = 1.6 \times 10^{13}[nm^{-1}s^{-1}]$ is the gain factor and $b = 5[nm^{-1}s^{-1}]$ is the amount of offset. If the spectral bandwidth of the spectrometer is 10nm, what is the total responsivity of the detector over the given wavelength range?



PART B) (2 pts.) How does this compare to the ideal responsivity?

Possibly Useful Information

$A = 2898$ $[\mu m\ K]$
$\sigma = 5.67 \times 10^{-8}$ $[W\ m^{-2}\ K^{-4}]$
$\mathrm{h} = 6.624 \times 10^{-34}$ $[J\ s]$
$\mathrm{c} = 3.0 \times 10^{8}$ $[m\ s^{-1}]$
$\mathrm{e} = 1.60217 \times 10^{-19}$ [coulombs]

9. **Radiometry (10 pts)**

PART A) (6 pts) On a warm day with a surrounding ambient temperature of 80.3 F (300 K), I decide to measure the side of the RIT dorm building in the LWIR, which acts as an ideal blackbody and is 800 meters away along a horizontal path. I have previously determined the transmission along this path to be 60%. While calibrating my radiometer, I obtain the following radiance values, when looking at a blackbody at various temperatures:

T [K]	L [$\cdot 10^{-5}\, W m^{-2} sr^{-1}$]
240	1
260	3
280	5
300	7
320	9

Knowing the temperature of the dorm building is 280 K, what temperature would this seem to look like at my sensor positioned 800 meters away? (That is, the *apparent* temperature, since my instrument doesn't really measure temperature, it actually measures radiance).

PART B) (4 pts) If I image the dorm building at a distance of 2000 meters, what would the apparent temperature be? Explain what caused the apparent temperature to change.

Possibly Useful Information
$A = 2898$ [$\mu m\ K$]
$\sigma = 5.67 \times 10^{-8}$ [$W\ m^{-2}\ K^{-4}$]
h $= 6.624 \times 10^{-34}$ [$J\ s$]
c $= 3.0 \times 10^{8}$ [$m\ s^{-1}$]
e $= 1.60217 \times 10^{-19}$ [coulombs]

10. **Fourier Methods in Imaging (10 pts)**

Consider the convolution of $N+1$ functions

$$g[x] = f_0[x] * f_1[x] * \cdots * f_N[x]$$

where the inputs are identically negative quadratic-phase factors with unit "chirp rate:"

$$f_n[x] = \exp\left[-i\pi\left(\frac{x}{1}\right)^2\right] = e^{-i\pi x^2}$$

(a) Find a single simple expression for $g[x]$

(b) Find a function $h[x]$ such that $g[x] * h[x] = \delta[x]$ and compare the expressions for $h[x]$ and $g[x]$.

11. **Fourier Methods in Imaging (10 pts)**

For each of the functions listed, where the symbols "$*$" and "$\bigstar$"represent the operations of "convolution" and "correlation," repectively, evaluate and/or simplify the three expressions, sketch the results of each, and evaluate and sketch the Fourier transforms of each

(a)

$$f_1[x] = \exp\left[+i\pi \cdot \frac{x^2}{2}\right] + \left(\delta\left[x + \frac{1}{2}\right] \cdot (-1 - i)\right)$$

(b)

$$f_2[x] = \exp\left[+i\pi \cdot \frac{x^2}{2}\right] \cdot \left(\delta\left[x + \frac{1}{2}\right] \cdot (-1 - i)\right)$$

(c)

$$f_3[x] = \exp\left[+i\pi \cdot \frac{x^2}{2}\right] * \left(\delta\left[x + \frac{1}{2}\right] \cdot (-1 - i)\right)$$

(d)

$$f_4[x] = \exp\left[+i\pi \cdot \frac{x^2}{2}\right] \bigstar \left(\delta\left[x + \frac{1}{2}\right] \cdot (-1 - i)\right)$$

12. **Fourier Methods in Imaging (10 pts)**

A one-dimensional continuous function $f[x]$ is to be sampled at uniform intervals separated by $\Delta x = 10\mu m$. At each sample, the signal is uniformly averaged over the spatial extent of the sensor, which is $d_0 = 10\mu m$. Assume ideal sampling, ideal sensor response, and an infinite number of samples.

(a) Write down the functional form of the sampled data for the 1-D function $f[x]$ and the system as described.

(b) Determine the maximum spatial frequency that may be sampled by this system without aliasing.

(c) Evaluate the MTF of the sampling system at the maximum sampling frequency just evaluated.

The suggestion is made that the maximum spatial frequency may be doubled for stationary functions (i.e., functions that do not move over time) by collecting a second set of samples after displacing the sensor by half of the sample spacing. In other words, a set of samples is collected using the configuration above. The sampling system (including the sensors) is then translated by the distance $\frac{\Delta x}{2} = 5\mu m$ and a second set of samples is collected. The second set of samples is interpolated between the original samples to obtain a data set with sample spacing of $5\mu m$. (There are real-world systems that follow this exact procedure).

(d) Explain the advantages and disadvantages of the new system for sampling of stationary functions. If there is a requirement to increase the maximum frequency of the sampling system, would you recommend such a procedure to your boss?

13. **Image Processing and Computer Vision (10 pts)**

Let $\mathcal{P} = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_m\}$ and $\mathcal{P}' = \{\mathbf{p}'_1, \mathbf{p}'_2, \ldots, \mathbf{p}'_m\}$ be a set of corresponding points that have been identified in two-dimensional floating and reference images, respectively. Each $\mathbf{p}_i$ and $\mathbf{p}'_i$ are 2-element column vectors defining x and y coordinates of the corresponding points; i.e., $\mathbf{p}_i = (x_i, y_i)^{\mathbf{T}}$ and $\mathbf{p}'_i = (x'_i, y'_i)^{\mathbf{T}}$.

Suppose we want to perform affine image registration to identify a 2×2 matrix $\mathbf{K} = \begin{bmatrix} k_{1,1} & k_{1,2} \\ k_{2,1} & k_{2,2} \end{bmatrix}$ and a 2×1 translation vector $\mathbf{t} = (t_1, t_2)^{\mathbf{T}}$ that relate points in the floating and reference images. If the corresponding points were perfectly described by an affine model, then we would have $\mathbf{p}'_i = \mathbf{K}\mathbf{p}_i + \mathbf{t}$ for all $i = 1, 2, \ldots, m$. However, we have some noise in our point correspondences. Thus, in order to perform affine registration, we solve the following minimization problem:

$$\min_{\mathbf{K},\mathbf{t}} \quad \sum_{i=1}^{m} \left\|\mathbf{p}'_i - (\mathbf{K}\mathbf{p}_i + \mathbf{t})\right\|^2 \ . \tag{1}$$

(a) Construct a $2m \times 6$ matrix $\mathbf{A}$, a 6×1 vector $\mathbf{v}$, and a $2m \times 1$ vector $\mathbf{w}$ in terms of the entries of $\mathbf{K}$ and $\mathbf{t}$ and the coordinates of the points in $\mathcal{P}$ and $\mathcal{P}'$, and show that (1) is equivalent to the following linear least squares problem:

$$\min_{\mathbf{v}} \quad (\mathbf{A}\mathbf{v} - \mathbf{w})^{\mathbf{T}} (\mathbf{A}\mathbf{v} - \mathbf{w}) \ . \tag{2}$$

(b) Show that the solution to (2) is $\mathbf{v} = \left(\mathbf{A}^{\mathbf{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathbf{T}}\mathbf{w}$.

14. **Image Processing and Computer Vision (10 pts)**

Suppose $f(\mathbf{x}) : \Omega \to \mathbb{R}$ is a noisy observed image, and suppose we want to identify a noise-free image $u(\mathbf{x}) : \Omega \to \mathbb{R}$ that is "close" to f. Consider the following functional minimization problem:

$$\min_u \quad \varepsilon(u) := \varepsilon_{\text{smooth}}(u) + \lambda \varepsilon_{\text{data}}(u) \quad ,$$

where

$$\varepsilon_{\text{smooth}}(u) = \int_\Omega \|\nabla u(\mathbf{x})\|^2 \, d\mathbf{x} \quad \text{and} \quad \varepsilon_{\text{data}}(u) = \int_\Omega (f(\mathbf{x}) - u(\mathbf{x}))^2 \, d\mathbf{x} \ .$$

Show that critical points of $\varepsilon(u)$ satisfy the Euler-Lagrange equation:

$$-\Delta u(\mathbf{x}) + \lambda \left(u(\mathbf{x}) - f(\mathbf{x}) \right) = 0 \ ,$$

ignoring boundary conditions.

15. **Image Processing and Computer Vision (10 pts)**

Let $\mathcal{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m\}$ be a set of m points in $\mathbb{R}^n$, and assume $m > n$. Assume we want to use Principal Components Analysis (PCA) to identify a low-dimensional representation of the points in $\mathcal{X}$ (that is, to construct a corresponding set of points $\mathcal{Y} = \{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_m\}$ in $\mathbb{R}^k$ with $k < n$). Describe the steps necessary for computing the points in $\mathcal{Y}$. Your description should provide enough detail so that an expert programmer can implement the construction of $\mathcal{Y}$ solely from your description.